

An $SU(5)$ grand unified model with discrete flavour symmetries.

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We propose a model based on the $SU(5)$ grand unification with an extra $Z_2 \otimes Z_2' \otimes Z_2'' \otimes Z_4 \otimes Z_{12}$ flavor symmetry, which successfully describes the observed SM fermion mass and mixing pattern. The observed quark mass and mixing pattern is caused by the Z_4 and Z_{12} symmetries, which are broken at very high scale by the $SU(5)$ scalar singlets σ and χ , charged respectively under these symmetries and which acquire VEVs at the GUT scale. The light neutrino masses are generated via a type I seesaw mechanism with three heavy Majorana neutrinos. The model has in total 17 effective free parameters, from which 2 are fixed and 15 are fitted to reproduce the experimental values of the 18 physical parameters in the quark and lepton sectors. The model predictions for both quark and lepton sectors are in excellent agreement with the experimental data.

I. INTRODUCTION

In spite of the Standard Model (SM) great success in describing electroweak phenomena, recently confirmed with the LHC discovery of a 126 GeV Higgs boson [1], it has many open questions [2, 3]. Among the most pressing are the smallness of neutrino masses, the fermion mass and mixing hierarchy, and the existence of the three generations of fermions. The existing pattern of fermion masses goes over a range of five orders of magnitude in the quark sector and a much wider range when neutrinos are included. While the mixing angles in the quark sector are very small, in the lepton sector two of the mixing angles are large, and one mixing angle is small. This suggests a different kind of New Physics for the neutrino sector from the one present in the quark mass and mixing pattern. Experiments with solar, atmospheric and reactor neutrinos have brought clear evidence of neutrino oscillations from the measured non vanishing neutrino mass squared splittings. This brings compelling and indubitable evidence that at least two of the neutrinos have non vanishing masses, much smaller, by many orders of magnitude, than the SM charged fermion masses, and that the three neutrino flavors mix.

The global fits of the available data from neutrino oscillation experiments Daya Bay [4], T2K [5], MINOS [6], Double CHOOZ [7] and RENO [8], constrain the neutrino mass squared splittings and mixing parameters, as shown in Tables I and II (based on Ref. [9]) for the normal (NH) and inverted (IH) hierarchies of the neutrino mass spectrum. These facts might suggest that the tiny neutrino masses can be related to a scale of New Physics that, in general, is not related to the scale of Electroweak Symmetry Breaking (EWSB) $v = 246$ GeV.

Parameter	$\Delta m_{21}^2 (10^{-5} \text{eV}^2)$	$\Delta m_{31}^2 (10^{-3} \text{eV}^2)$	$(\sin^2 \theta_{12})_{\text{exp}}$	$(\sin^2 \theta_{23})_{\text{exp}}$	$(\sin^2 \theta_{13})_{\text{exp}}$
Best fit	7.60	2.48	0.323	0.567	0.0234
1σ range	7.42 – 7.79	2.41 – 2.53	0.307 – 0.339	0.439 – 0.599	0.0214 – 0.0254
2σ range	7.26 – 7.99	2.35 – 2.59	0.292 – 0.357	0.413 – 0.623	0.0195 – 0.0274
3σ range	7.11 – 8.11	2.30 – 2.65	0.278 – 0.375	0.392 – 0.643	0.0183 – 0.0297

Table I: Range for experimental values of neutrino mass squared splittings and leptonic mixing parameters, taken from Ref. [9], for the case of normal hierarchy.

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Parameter	$\Delta m_{21}^2 (10^{-5} \text{eV}^2)$	$\Delta m_{13}^2 (10^{-3} \text{eV}^2)$	$(\sin^2 \theta_{12})_{\text{exp}}$	$(\sin^2 \theta_{23})_{\text{exp}}$	$(\sin^2 \theta_{13})_{\text{exp}}$
Best fit	7.60	2.38	0.323	0.573	0.0240
1σ range	7.42 – 7.79	2.32 – 2.43	0.307 – 0.339	0.530 – 0.598	0.0221 – 0.0259
2σ range	7.26 – 7.99	2.26 – 2.48	0.292 – 0.357	0.432 – 0.621	0.0202 – 0.0278
3σ range	7.11 – 8.11	2.20 – 2.54	0.278 – 0.375	0.403 – 0.640	0.0183 – 0.0297

Table II: Range for experimental values of neutrino mass squared splittings and leptonic mixing parameters, taken from Ref. [9], for the case of inverted hierarchy.

The flavour puzzle of the SM indicates that New Physics has to be advocated to explain the prevailing pattern of fermion masses and mixings. To tackle the limitations of the SM, various extensions of the SM including larger scalar and/or fermion sector as well as extended gauge group with additional flavor symmetries, have been proposed in the literature (for a reviews see, e.g., Refs. [10–14]). Another approach to describe the fermion mass and mixing pattern consists in postulating particular mass matrix textures (see Ref [15] for some works considering textures). Concerning models with an extended gauge symmetry, Grand unified theories (GUTs) endowed with flavor symmetries may provide an unified description for the mass and mixing pattern of leptons and quarks. This is motivated by the fact that leptons and quarks belong to the same multiplets of the GUT group, allowing to relate their masses and mixings [16, 17]. This framework is also very useful for explaining the smallness of neutrino masses through the simplest type I seesaw mechanism, where the new heavy Majorana neutrinos have masses at the GUT scale. Various GUT models with flavor symmetries have been proposed in the literature [18–29]. For a general review see for example [30, 31].

Recently we proposed a model based on the $SU(5)$ grand unification with an extra $A_4 \otimes Z_2 \otimes Z'_2 \otimes Z''_2 \otimes U(1)_f$ flavor symmetry [32], which successfully accounts for the SM fermion mass and mixing pattern. That model involves a horizontal symmetry $U_f(1)$, which provides an explanation for the prevailing pattern of charged fermion masses and quark mixing matrix elements, by means of generalized Froggatt-Nielsen mechanism [33]. In that model, the light neutrino masses are generated via a radiative seesaw mechanism, with a single heavy Majorana neutrino and neutral scalars running in the loops. Nevertheless, that model has a non minimal scalar sector, and at low energies reduces to an eight Higgs doublet model (8HDM) with a light scalar octet, thus making it not predictive in the scalar sector. Furthermore, in that model the obtained values for the observables in the quark and lepton sector are in good agreement with the experimental data, with the exception of the up and charm quark masses.

It is interesting to find an alternative and better explanation for the SM fermion mass and mixing hierarchy, by formulating a $SU(5)$ grand unification with less scalar content than our previous model of Ref. [32]. To this end, we propose an alternative and improved version of the $SU(5)$ GUT model with an additional flavor symmetry group $Z_2 \otimes Z'_2 \otimes Z''_2 \otimes Z_4 \otimes Z_{12}$, which is consistent with the current data on fermion masses and mixings. The particular role of each discrete symmetry is explained in the following. The Z_2 symmetry separates the scalars participating in the Yukawa interactions for charged leptons and down type quarks from those ones participating in the Yukawa interactions for up type quarks. This results in a separation of the up type quark sector from the down type quark and charged lepton sector, thus reducing the number of model parameters. The Z'_2 symmetry determines the allowed entries of the mass matrices for down type quarks, charged leptons and neutrinos. The Z''_2 symmetry separates the heavy right handed Majorana neutrinos from the remaining fermionic fields. The Z_4 symmetry is crucial for explaining the smallness of the down quark and electron masses. Without this symmetry they both would be larger than their corresponding experimental values by about two orders of magnitude, unless one sets their Yukawa couplings unnaturally small. The Z_{12} symmetry induces most of the charged fermion mass and quark mixing hierarchy. Let us recall that due to the properties of the Z_N groups, it follows that Z_{12} is the lowest cyclic symmetry that allows to build a ten dimensional up type quark Yukawa term, crucial to get the required λ^6 suppression in the 11 entry of the up type quark mass matrix, where $\lambda = 0.225$ is one of the Wolfenstein parameters. This symmetry is essential in order to get the observed pattern of charged fermion masses and quark mixings. The scalar sector of our model includes the following $SU(5)$ representations: one **24**, one **45**'s, three **5**'s and four **1**'s. The four $SU(5)$ scalar singlets and the scalar in the **24** irrep of $SU(5)$ acquire vacuum expectation values (VEVs) at the GUT scale. The particular role of each additional scalar field and the corresponding particle assignments under the symmetry group of the model are explained in detail in Sec. II. In the present model the fermion sector is extended by introducing three heavy right handed Majorana neutrinos, which are singlets under the SM group. These heavy right handed Majorana neutrinos, which acquire masses at the GUT scale due to their interactions with the $SU(5)$ singlet scalar fields, allow us to generate small active neutrino masses through type I seesaw mechanism. In this framework, the active neutrinos acquire small masses scaled by the inverse of the large Majorana neutrino masses, thus providing a natural explanation for the smallness of neutrino masses.

Our model at low energies corresponds to a four Higgs doublet model (4HDM), with a light scalar color octet and is more minimal than several models proposed in the literature, such as [23–25, 32]. Our model successfully describes

a realistic pattern of the SM fermion masses and mixings. The model has 16 free effective parameters, from which 2 are fixed and 14 are fitted to reproduce the experimental values of 18 observables, i.e., 9 charged fermion masses, 2 neutrino mass squared splittings, 3 lepton mixing parameters, 3 quark mixing angles and 1 CP violating phase of the CKM quark mixing matrix. All obtained physical parameters in the quark and lepton sector are in excellent agreement with the experimental data.

The paper is organized as follows. In Sec. II we explain theoretical aspects of the proposed model. In Sec. III we focus on the discussion of quark masses and mixing and give our corresponding results. Our results regarding lepton masses and mixing, followed by a numerical analysis, are presented in Sec. IV. Conclusions are given in Sec. V.

II. THE MODEL

As is well known, the minimal $SU(5)$ GUT [34] with fermions in $\bar{\mathbf{5}} + \mathbf{10}$ and the scalars in $\mathbf{5} + \mathbf{24}$ representations of $SU(5)$, has several drawbacks. In particular, it predicts wrong relations between the down-type quark and charged lepton masses, short proton life-time, and the unification of gauge couplings is not consistent with the values of α_S , $\sin\theta_W$ and α_{em} at the M_Z scale. The minimal model does not account for non vanishing neutrino masses, contradicting neutrino oscillation experiments. Addressing some of these problems requires an extension of the model scalar sector, by including, in particular, a scalar $\mathbf{45}$ representation of $SU(5)$ [35–47]. However, the next-to-minimal $SU(5)$ GUT model is unsatisfactory in describing the fermion mass and mixing pattern, due to the unexplained hierarchy among the large number of Yukawa couplings in the model. To address that problem, we recently proposed a model based on the $SU(5)$ grand unification with an extra $A_4 \otimes Z_2 \otimes Z'_2 \otimes Z''_2 \otimes U(1)_f$ flavor symmetry, which successfully accounts for the SM fermion mass and mixing pattern. In that model the fermion mass hierarchy is explained by a spontaneously broken group $U(1)_f$ with a special $U(1)_f$ charge assignment to the fields participating in the Yukawa terms. However, that model has a non minimal scalar sector, and at low energies reduces to an eight Higgs doublet model with a light scalar octet, which is not predictive in the scalar sector. Therefore it would be desirable to explain the SM fermion mass and mixing hierarchy by formulating a $SU(5)$ grand unification model with a more minimal scalar content than our previous model of Ref. [32]. To this end, we consider a multi-Higgs extension of the next-to-minimal $SU(5)$ GUT, with the full symmetry \mathcal{G} experiencing a two-step spontaneous breaking:

$$\begin{aligned}
 \mathcal{G} &= SU(5) \otimes Z_2 \otimes Z'_2 \otimes Z''_2 \otimes Z_4 \otimes Z_{12} \\
 &\Downarrow \Lambda_{GUT} \\
 &SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes Z_2 \otimes Z'_2 \\
 &\Downarrow \Lambda_{EW} \\
 &SU(3)_C \otimes U(1)_{em}
 \end{aligned} \tag{1}$$

A relevant difference of this Grand Unified Model with our previous $SU(5)$ model with A_4 flavour symmetry is that the former does not involve neither the global $U(1)$ symmetry crucial to trigger the generalized Froggatt-Nielsen mechanism nor the A_4 flavour symmetry while the later does. Our current $SU(5)$ GUT model involves instead a set of Z_2 , Z'_2 , Z''_2 , Z_4 and Z_{12} discrete symmetries. Furthermore, our $SU(5)$ GUT model with A_4 flavour symmetry of Ref. [32] includes the Z_2 , Z'_2 and Z''_2 discrete symmetries, with a unbroken Z_2 symmetry, while in our current $SU(5) \otimes Z_2 \otimes Z'_2 \otimes Z''_2 \otimes Z_4 \otimes Z_{12}$ GUT model, the Z_2 symmetry is broken as well as the remaining discrete symmetries. While the linear combinations of $U(1)$ charges of the fields participating in the Yukawa terms are put by hand in our $SU(5)$ GUT model with A_4 flavour symmetry in order to generate specific fermion mass matrix textures, in our current $SU(5)$ GUT model the hierarchy among the fermion masses naturally arises from the $Z_4 \otimes Z_{12}$ charge assignments of the fermion and scalar fields. In particular, the Z_{12} symmetry together with the Z_4 symmetries will be crucial for explaining the smallness of the first generation charged fermions. Besides that, the $Z_4 \otimes Z_{12}$ symmetry will shape the hierarchical structure of the up and down type quark mass matrices necessary to get a realistic pattern of quark masses and mixings. The Z_2 symmetry will separate the up type quark sector from the down type quark and charged lepton sector resulting in a reduction of the number of model parameters. Let us recall that due to the properties of the Z_N groups, it follows that Z_{12} is the lowest cyclic symmetry that allows to build a ten dimensional up type quark Yukawa term, crucial to get the required λ^6 suppression in the 11 entry of the up type quark mass matrix, where $\lambda = 0.225$ is one of the Wolfenstein parameters. The Z'_2 symmetry will determine the allowed entries of the mass matrices for down type quarks and charged leptons. The Z''_2 symmetry separates the heavy right handed Majorana neutrinos from the remaining fermionic fields. The Z_4 symmetry is crucial for explaining the smallness of the down quark and electron masses. Without this symmetry they both would be larger by about two orders of magnitude

than their corresponding experimental values, unless one sets the corresponding Yukawa couplings unnaturally small. Furthermore, while the CKM matrix is fitted to the experimental data in the $SU(5)$ GUT model with A_4 flavour symmetry, our current $SU(5)$ GUT model predicts a specific hierarchical structure for the CKM matrix, consistent with the experimental data. All the aforementioned features make our current model an important improvement of our previous $SU(5)$ GUT model with A_4 flavour symmetry of Ref. [32]. In the present model the fermion sector is extended by introducing three heavy right handed Majorana neutrinos, which are singlets under the SM group. The fermion assignments under the group $\mathcal{G} = SU(5) \otimes Z_2 \otimes Z'_2 \otimes Z''_2 \otimes Z_4 \otimes Z_{12}$ are:

$$\begin{aligned} \psi^{i(1)} &\sim (\bar{\mathbf{5}}, -1, -1, 1, -1, -i), & \psi^{i(2)} &\sim (\bar{\mathbf{5}}, -1, 1, 1, 1, -i), & \psi^{i(3)} &\sim (\bar{\mathbf{5}}, -1, 1, 1, 1, -i), \\ \Psi_{ij}^{(1)} &\sim (\mathbf{10}, 1, 1, 1, 1, i), & \Psi_{ij}^{(2)} &\sim (\mathbf{10}, 1, 1, 1, 1, \omega), & \Psi_{ij}^{(3)} &\sim (\mathbf{10}, 1, 1, 1, 1, 1), & i, j = 1, 2, 3, 4, 5. \\ N_R^{(1)} &\sim (\mathbf{1}, -1, -1, -1, 1, 1), & N_R^{(2)} &\sim (\mathbf{1}, -1, 1, -1, 1, 1), & N_R^{(3)} &\sim (\mathbf{1}, -1, 1, -1, 1, -1) \end{aligned} \quad (2)$$

where $\omega = e^{\frac{2\pi i}{3}}$.

More explicitly, we accomodate the fermions as follows:

$$\Psi_{ij}^{(f)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u_3^{(f)c} & -u_2^{(f)c} & -u_1^{(f)} & -d_1^{(f)} \\ -u_3^{(f)c} & 0 & u_1^{(f)c} & -u_2^{(f)} & -d_2^{(f)} \\ u_2^{(f)c} & -u_1^{(f)c} & 0 & -u_3^{(f)} & -d_3^{(f)} \\ u_1^{(f)} & u_2^{(f)} & u_3^{(f)} & 0 & -l^{(f)c} \\ d_1^{(f)} & d_2^{(f)} & d_3^{(f)} & l^{(f)c} & 0 \end{pmatrix}_L, \quad f = 1, 2, 3 \quad i, j = 1, 2, 3, 4, 5. \quad (3)$$

$$\psi^{i(f)} = \left(d_1^{(f)c}, d_2^{(f)c}, d_3^{(f)c}, l^{(f)}, -\nu_f \right)_L. \quad (4)$$

Where the subindices denote the different quark colors whereas the superscript f labels the fermion families.

The scalar sector is composed of the following $SU(5)$ representations: one **24**, one **45**'s, three **5**'s and four **1**'s. Thus, the scalar fields of our model have the following \mathcal{G} assignments:

$$\begin{aligned} \chi &\sim (\mathbf{1}, 1, 1, 1, 1, \omega^{\frac{1}{4}}), & \sigma &\sim (\mathbf{1}, 1, 1, 1, 1, i), & \eta &\sim (\mathbf{1}, 1, 1, -1, -1, 1) \\ \zeta &\sim (\mathbf{1}, 1, 1, -1, 1, 1), & H_i^{(1)} &\sim (\mathbf{5}, 1, 1, 1, 1, 1), & H_i^{(2)} &\sim (\mathbf{5}, 1, -1, 1, 1, 1), \\ H_i^{(3)} &\sim (\mathbf{5}, -1, 1, 1, 1, 1), & \Sigma_j^i &\sim (\mathbf{24}, 1, 1, 1, 1, 1), & \Phi_{jk}^i &\sim (\mathbf{45}, -1, 1, 1, 1, 1). \end{aligned} \quad (5)$$

Note that the aforementioned scalar content of our model is much more minimal than the corresponding to our previous model of $SU(5)$ GUT model with A_4 flavour symmetry of Ref. [32], which includes one **24**, one **45**, seven **5**'s and six **1**'s irreps of $SU(5)$. As previously mentioned, the scalar field Σ gets a vacuum expectation value (VEV) at the GUT scale $\Lambda_{GUT} = 10^{16}$ GeV and triggers the first step of symmetry breaking in Eq. (1). This first step is also induced by the scalars χ , σ , η and ζ , which get VEVs at the GUT scale. The second step of symmetry breaking, is caused by the scalars $H_i^{(f)}$ ($f = 1, 2, 3$) and Φ_{jk}^i acquiring VEVs at the scale of electroweak symmetry breaking.

Our current model is based on the following assumptions:

1. The symmetry of the $SU(5)$ GUT Model is extended to include the discrete symmetries Z_2 , Z'_2 , Z''_2 , Z_4 and Z_{12} . The Z_2 , Z'_2 and Z''_2 , Z_4 , Z_{12} discrete symmetries are broken at the Electroweak and GUT scales, respectively.
2. The scalar sector includes the following $SU(5)$ representations: one **24**, one **45**'s, three **5**'s and four **1**'s. The four $SU(5)$ scalar singlets and the scalar in the **24** irrep of $SU(5)$ acquire VEVs at the GUT scale. The scalar field in the **24** irrep of $SU(5)$ is needed to trigger the first step of symmetry breaking in Eq. (1), which is also induced by the **1**'s irreps of $SU(5)$. The remaining scalars acquire VEVs at the electroweak scale and induce the second step of symmetry breaking. As previously mentioned, having scalar fields in the **45** representation of $SU(5)$ is crucial to get the correct mass relations of down-type quarks and charged leptons.
3. The Z_2 symmetry separates the scalars in the **5** and **45** irreps of $SU(5)$ participating in the Yukawa interactions for charged leptons and down type quarks from those ones participating in the Yukawa interactions for up type quarks. This implies the $SU(5)$ scalar multiplets contributing to the masses of the down-type quarks and charged leptons are different from those ones providing masses to the up-type quarks. The fermions belonging

to the **10** irrep of $SU(5)$ are Z_2 even while those ones embedding in the $\bar{\mathbf{5}}$ irrep of $SU(5)$ are Z_2 odd. The **45** and one of the **5**'s scalars are Z_2 odd and thus they participate in the Yukawa interactions for charged leptons and down type quarks. The remaining two **5**'s, which are Z_2 even participate in the Yukawa interactions for up type quarks. The three scalars $SU(5)$ singlets are Z_2 even.

4. The Z'_2 symmetry determines the allowed entries of the mass matrices for down type quarks, charged leptons and neutrinos. The Z'_2 symmetry separates the Z'_2 odd fermionic $\bar{\mathbf{5}}^{(1)}$ irrep of $SU(5)$ belonging to the first family from the remaining fermionic $\bar{\mathbf{5}}^{(2)}$ and $\bar{\mathbf{5}}^{(3)}$ irreps of $SU(5)$, neutral under this symmetry. Furthermore, the Z'_2 symmetry separates the first generation right handed heavy Majorana neutrino $N_R^{(1)}$, neutral under this symmetry, from the second and third generation ones, i.e., $N_R^{(2)}$ and $N_R^{(3)}$, charged under the Z'_2 symmetry. Thus, the Z'_2 symmetry forbids mixings of the first generation right handed heavy Majorana neutrino $N_R^{(1)}$ with the second and third generation ones, i.e., $N_R^{(2)}$ and $N_R^{(3)}$. This symmetry also distinguishes the $SU(5)$ quintuplets $H_i^{(2)}$ charged under this symmetry from the remaining $SU(5)$ quintuplets $H_i^{(1)}$, $H_i^{(3)}$, neutral under this symmetry.
5. The Z''_2 symmetry distinguishes the right handed heavy Majorana neutrinos, odd under this symmetry from the remaining fermionic fields, even under Z''_2 . In the scalar sector, only η and ζ are odd under this symmetry, while the remaining scalars are Z''_2 even.
6. The Z_4 symmetry separates the fermionic $\bar{\mathbf{5}}^{(1)}$ irrep of $SU(5)$ belonging to the first family from the remaining fermionic $\bar{\mathbf{5}}$ irreps of $SU(5)$, neutral under this symmetry. This Z_4 symmetry also distinguishes the $SU(5)$ scalar singlets σ and η charged under Z_4 from the remaining scalar fields, neutral under this symmetry. Furthermore, it is assumed that the right handed heavy Majorana neutrino are charged under Z_4 symmetry. Without the Z_4 charged $SU(5)$ scalar singlet σ , the down quark and electron masses would be larger by about two orders of magnitude than their corresponding experimental values, unless one sets the corresponding Yukawa couplings unnaturally small. It is noteworthy, that unlike in the up type quark sector, a λ^8 suppression in the 11 entry of the mass matrices for down type quarks and charged leptons is required to naturally explain the smallness of the down quark and electron masses. The Z_4 and the Z_{12} symmetries will be crucial to achieve that λ^8 suppression, where $\lambda = 0.225$ is one of the Wolfenstein parameters.
7. The Z_{12} symmetry shapes the hierarchical structure of the quark mass matrices necessary to get a realistic pattern of quark masses and mixings. Besides that, the charged lepton mass hierarchy also arises from Z_{12} symmetry. Let us recall that due to the properties of the Z_N groups, it follows that Z_{12} is the lowest cyclic symmetry that allows to build a ten dimensional up type quark Yukawa term with a $\frac{\lambda^6}{\lambda^6}$ insertion in a four dimensional term, crucial to get the required λ^6 suppression in the 11 entry of the up type quark mass matrix. This symmetry distinguishes the fermionic **10**⁽³⁾ irrep of $SU(5)$ corresponding to the third family, i.e., $\Psi_{ij}^{(3)}$, neutral under Z_{12} from the remaining fermionic fields, charged under this symmetry. It is assumed that all fermionic $\bar{\mathbf{5}}^{(f)}$ irreps of $SU(5)$ ($f = 1, 2, 3$) have the same Z_{12} charges, different from the Z_{12} charge of $\bar{\mathbf{5}}^{(1)}$. All scalars are neutral under the Z_{12} symmetry, except the $SU(5)$ scalar singlet χ . The Z_{12} symmetry strongly suppresses mixings of the third generation right handed heavy Majorana neutrino $N_R^{(3)}$, charged under this symmetry, with the first and second generation ones $N_R^{(1)}$ and $N_R^{(2)}$, which are Z_{12} neutral. Note that the heavy Majorana neutrino $N_R^{(3)}$ is the only fermion which is assumed to be charged under Z_{12} .

We consider the following VEV pattern of the scalars fields of the model. The VEVs of the scalars $H_i^{(f)}$ ($f = 1, 2, 3$) and Σ_j^i are given by:

$$\langle H_i^{(f)} \rangle = v_H^{(f)} \delta_{i5}, \quad \langle \Sigma_j^i \rangle = v_\Sigma \text{diag} \left(1, 1, 1, -\frac{3}{2}, -\frac{3}{2} \right), \quad f = 1, 2, 3, \quad (6)$$

Furthermore, the VEV pattern for the Σ field given above, which is consistent with the minimization conditions of the scalar potential, follows from the general group theory of spontaneous symmetry breakdown, as shown in Ref. [48].

Assuming that the hierarchy of charged fermion masses and quark mixing matrix elements is explained by the Z_4 and Z_{12} symmetries, and in order to relate the quark masses with the quark mixing parameters, we set the VEVs of the $SU(5)$ scalar singlets as follows:

$$v_\eta \sim v_\zeta \sim v_\chi = v_\sigma = \Lambda_{GUT} = \lambda \Lambda, \quad (7)$$

where $\lambda = 0.225$ is one of the parameters in the Wolfenstein parametrization and Λ corresponds to the cutoff of our model.

From the properties of the **45** dimensional irrep of $SU(5)$, it follows that Φ_{jk}^i satisfies the following relations [35, 36]:

$$\Phi_{jk}^i = -\Phi_{kj}^i, \quad \sum_{i=1}^5 \Phi_{ij}^i = 0, \quad i, j, k = 1, 2, \dots, 5. \quad (8)$$

This results in the following only allowed non-zero VEVs of Φ_{jk}^i :

$$\langle \Phi_{p5}^p \rangle = -\frac{1}{3} \langle \Phi_{45}^4 \rangle = v_\Phi, \quad \langle \Phi_{j5}^i \rangle = v_\Phi (\delta_j^i - 4\delta_4^i \delta_j^4), \quad i, j = 1, 2, 3, 4, 5, \quad p = 1, 2, 3, 5. \quad (9)$$

With the above particle content, the relevant Yukawa terms invariant under the group \mathcal{G} are:

$$\begin{aligned} \mathcal{L}_Y = & \alpha_{11} \psi^{i(1)} H^{j(3)} \Psi_{ij}^{(1)} \frac{\chi^6 \sigma^2}{\Lambda^8} + \beta_{11} \psi^{i(1)} \Phi_i^{jk} \Psi_{jk}^{(1)} \frac{\chi^6 \sigma^2}{\Lambda^8} + \alpha_{22} \psi^{i(2)} H^{j(3)} \Psi_{ij}^{(2)} \frac{\chi^5}{\Lambda^5} + \beta_{22} \psi^{i(2)} \Phi_i^{jk} \Psi_{jk}^{(2)} \frac{\chi^5}{\Lambda^5} \\ & + \alpha_{23} \psi^{i(3)} H^{j(3)} \Psi_{ij}^{(2)} \frac{\chi^5}{\Lambda^5} + \beta_{23} \psi^{i(3)} \Phi_i^{jk} \Psi_{jk}^{(2)} \frac{\chi^5}{\Lambda^5} + \alpha_{32} \psi^{i(2)} H^{j(3)} \Psi_{ij}^{(3)} \frac{\chi^3}{\Lambda^3} + \alpha_{32} \psi^{i(2)} \Phi_i^{jk} \Psi_{jk}^{(3)} \frac{\chi^3}{\Lambda^3} \\ & + \alpha_{33} \psi^{i(3)} H^{j(3)} \Psi_{ij}^{(3)} \frac{\chi^3}{\Lambda^3} + \beta_{33} \psi^{i(3)} \Phi_i^{jk} \Psi_{jk}^{(3)} \frac{\chi^3}{\Lambda^3} \\ & + \varepsilon^{ijklp} \left\{ \gamma_{11} \Psi_{ij}^{(1)} H_p^{(1)} \Psi_{kl}^{(1)} \frac{\chi^6}{\Lambda^6} + \gamma_{22} \Psi_{ij}^{(2)} H_p^{(1)} \Psi_{kl}^{(2)} \frac{\chi^4}{\Lambda^4} + \gamma_{33} \Psi_{ij}^{(3)} H_p^{(1)} \Psi_{kl}^{(3)} \right. \\ & + \gamma_{12} \Psi_{ij}^{(1)} H_p^{(1)} \Psi_{kl}^{(2)} \frac{\chi^5}{\Lambda^5} + \gamma_{21} \Psi_{ij}^{(2)} H_p^{(1)} \Psi_{kl}^{(1)} \frac{\chi^5}{\Lambda^5} + \gamma_{13} \Psi_{ij}^{(1)} H_p^{(1)} \Psi_{kl}^{(3)} \frac{\chi^3}{\Lambda^3} \\ & + \gamma_{31} \Psi_{ij}^{(3)} H_p^{(1)} \Psi_{kl}^{(1)} \frac{\chi^3}{\Lambda^3} + \gamma_{23} \Psi_{ij}^{(2)} H_p^{(2)} \Psi_{kl}^{(3)} \frac{\chi^2}{\Lambda^2} + \gamma_{32} \Psi_{ij}^{(3)} H_p^{(2)} \Psi_{kl}^{(2)} \frac{\chi^2}{\Lambda^2} \left. \right\} \\ & + \left(y_1 \overline{N}_R^{(1)} N_R^{(1)c} + y_2 \overline{N}_R^{(2)} N_R^{(2)c} + y_3 \overline{N}_R^{(3)} N_R^{(3)c} \right) \frac{\chi^* \chi + x_1 \sigma^* \sigma + x_2 \eta^2 + x_3 \zeta^2}{\Lambda} \\ & + \varepsilon_{11} \psi^{i(1)} H_i^{(1)} N_R^{(1)} \frac{\chi^3 \eta}{\Lambda^4} + \varepsilon_{21} \psi^{i(2)} H_i^{(2)} N_R^{(1)} \frac{\chi^3 \zeta}{\Lambda^4} + \varepsilon_{31} \psi^{i(3)} H_i^{(2)} N_R^{(1)} \frac{\chi^3 \zeta}{\Lambda^4} \\ & + \varepsilon_{12} \psi^{i(1)} H_i^{(2)} N_R^{(2)} \frac{\chi^3 \eta}{\Lambda^4} + \varepsilon_{22} \psi^{i(2)} H_i^{(1)} N_R^{(2)} \frac{\chi^3 \zeta}{\Lambda^4} + \varepsilon_{32} \psi^{i(3)} H_i^{(1)} N_R^{(2)} \frac{\chi^3 \zeta}{\Lambda^4} \end{aligned} \quad (10)$$

where the dimensionless couplings in Eq. (10) are $\mathcal{O}(1)$ parameters. It is worth mentioning that the terms in the first, second and third lines of Eq. (10) contribute to the masses of the down-type quarks and charged leptons, the terms of the fourth, fifth and sixth lines of Eq. (10) give contributions to the up-type quark masses, while the remaining terms generate the neutrino masses. The aforementioned Yukawa terms do not include the interactions involving the third generation right handed heavy Majorana neutrino $N_R^{(3)}$, since they are strongly suppressed by powers of $\frac{v_\Phi^6}{\Lambda^6} = \lambda^6$, where $\lambda = 0.225$ is one of the Wolfenstein parameters. This is a consequence of the nontrivial Z_{12} charge assignment for the third generation right handed heavy Majorana neutrino. The lightest of the physical neutral scalar states of $H^{(1)}$, $H^{(2)}$, $H^{(3)}$ and Φ_{jk}^i is the SM-like 126 GeV Higgs discovered at the LHC [1]. Besides that, the resulting low energy effective theory corresponds to a four Higgs doublet model (4HDM), with a light scalar color octet. Note that our model is much more economical than our previous $SU(5)$ GUT model with A_4 flavour symmetry of Ref. [32], whose low energy effective theory corresponds to an eight Higgs doublet model (8HDM), with a light scalar octet. As we will show in section III, the dominant contribution to the top quark mass mainly arises from $H^{(1)}$. The SM-like 126 GeV Higgs also receives its main contributions from the CP even neutral state of the $SU(2)$ doublet part of $H^{(1)}$. The remaining scalars are heavy and outside the LHC reach. The large number of free uncorrelated parameters in the scalar potential allows us to adjust the required pattern of scalar masses. Therefore, by an appropriate choice of the free parameters in the scalar potential, one can suppress the loop effects of the heavy scalars contributing to certain observables.

III. QUARK MASSES AND MIXING

From Eq. (10) we get the following mass matrix textures for quarks:

$$M_U = \begin{pmatrix} a_{11}^{(U)} \lambda^6 & a_{12}^{(U)} \lambda^5 & a_{13}^{(U)} \lambda^3 \\ a_{12}^{(U)} \lambda^5 & a_{22}^{(U)} \lambda^4 & a_{23}^{(U)} \lambda^2 \\ a_{13}^{(U)} \lambda^3 & a_{23}^{(U)} \lambda^2 & a_{33}^{(U)} \end{pmatrix} \frac{v}{\sqrt{2}}, \quad (11)$$

$$M_D = \begin{pmatrix} a_{11}^{(D)} \lambda^8 & 0 & 0 \\ 0 & a_{22}^{(D)} \lambda^5 & a_{23}^{(D)} \lambda^5 \\ 0 & a_{32}^{(D)} \lambda^3 & a_{33}^{(D)} \lambda^3 \end{pmatrix} \frac{v}{\sqrt{2}}, \quad (12)$$

Furthermore, the $\mathcal{O}(1)$ dimensionless couplings in Eqs. (11) and (12) are given by the following relations:

$$\begin{aligned} a_{12}^{(U)} &= 2\sqrt{2}(\gamma_{12} + \gamma_{21}) \frac{v_H^{(2)}}{v}, & a_{11}^{(U)} &= 4\sqrt{2}\gamma_{11} \frac{v_H^{(1)}}{v}, & a_{13}^{(U)} &= 2\sqrt{2}(\gamma_{13} + \gamma_{31}) \frac{v_H^{(1)}}{v}, \\ a_{23}^{(U)} &= 2\sqrt{2}(\gamma_{23} + \gamma_{32}) \frac{v_H^{(1)}}{v}, & a_{22}^{(U)} &= 4\sqrt{2}\gamma_{22} \frac{v_H^{(1)}}{v}, & a_{33}^{(U)} &= 4\sqrt{2}\gamma_{33} \frac{v_H^{(1)}}{v}, \\ a_{23}^{(D)} &= \frac{1}{v}(\alpha_{23}v_H^{(3)} + 2\beta_{23}v_\Phi), & a_{11}^{(D)} &= \frac{1}{v}(\alpha_{11}v_H^{(3)} + 2\beta_{11}v_\Phi), & a_{22}^{(D)} &= \frac{1}{v}(\alpha_{22}v_H^{(3)} + 2\beta_{22}v_\Phi), \\ a_{32}^{(D)} &= \frac{1}{v}(\alpha_{32}v_H^{(3)} + 2\beta_{32}v_\Phi), & a_{33}^{(D)} &= \frac{1}{v}(\alpha_{33}v_H^{(3)} + 2\beta_{33}v_\Phi). \end{aligned} \quad (13)$$

Assuming that the hierarchy of charged fermion masses and quark mixing matrix elements is explained by the Z_{12} symmetry, we adopt an approximate universality in the dimensionless Yukawa couplings for up type quarks, down type quarks and charged leptons:

$$\begin{aligned} \gamma_{11} &= \gamma_1, & \gamma_{12} &= \gamma_{21} = \gamma_1 \left(1 - \frac{\lambda^2}{2}\right)^2, & \gamma_{22} &= \gamma_1 \left(1 - \frac{\lambda^2}{2}\right)^3, & \gamma_{13} &= \gamma_{31} = -\gamma_2 e^{-i\phi}, \\ \gamma_{23} &= \gamma_{32} = -\gamma_2 \left(1 - \frac{\lambda^2}{2}\right) e^{-i\phi}, & \gamma_{33} &= \gamma_4 e^{-2i\phi}, & \alpha_{ii} &= \alpha_i, & \beta_{ii} &= \beta_i, \\ \alpha_{ij} &= \tilde{\alpha}, & \beta_{ij} &= \tilde{\beta}, & i &\neq j, & i, j &= 1, 2, 3. \end{aligned} \quad (14)$$

where $\lambda = 0.225$, with $\gamma_1, \gamma_2, \tilde{\alpha}, \tilde{\beta}, \alpha_i$ and β_i ($i = 1, 2, 3$) real $\mathcal{O}(1)$ parameters. Note that exact universality in the dimensionless quark Yukawa couplings leads to massless up, charm and strange quarks. Consequently a breaking of universality in the quark Yukawa couplings is required to generate these masses. Furthermore, for the sake of simplicity, we assume that the complex phase responsible for CP violation in the quark sector only arises from the up type quark sector. In addition, to simplify the analysis, we set $v_H^{(1)} = v_H^{(2)}$ and we fix $|a_{33}^{(U)}| = 1$, as suggested by the naturalness arguments. Therefore, the up and down type quark mass matrices take the following form:

$$M_U = P^\dagger \widetilde{M}_U P^\dagger, \quad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\phi} \end{pmatrix},$$

$$\widetilde{M}_U = \begin{pmatrix} a_1^{(U)} \lambda^6 & a_1^{(U)} \left(1 - \frac{\lambda^2}{2}\right)^2 \lambda^5 & -a_2^{(U)} \lambda^3 \\ a_1^{(U)} \left(1 - \frac{\lambda^2}{2}\right)^2 \lambda^5 & a_1^{(U)} \left(1 - \frac{\lambda^2}{2}\right)^3 \lambda^4 & -a_2^{(U)} \left(1 - \frac{\lambda^2}{2}\right) \lambda^2 \\ -a_2^{(U)} \left(1 - \frac{\lambda^2}{2}\right) \lambda^3 & -a_2^{(U)} \left(1 - \frac{\lambda^2}{2}\right) \lambda^2 & 1 \end{pmatrix} \frac{v}{\sqrt{2}}, \quad (15)$$

$$M_D = \begin{pmatrix} a_1^{(D)} \lambda^8 & 0 & 0 \\ 0 & a_2^{(D)} \lambda^5 & a_4^{(D)} \lambda^5 \\ 0 & a_4^{(D)} \lambda^3 & a_3^{(D)} \lambda^3 \end{pmatrix} \frac{v}{\sqrt{2}}. \quad (16)$$

From Eqs. (15) and (16) it follows that the up and down type quark masses are approximately given by:

$$\begin{aligned}
m_u &\simeq \frac{a_1^{(U)}}{2} \left(1 - \frac{\lambda^2}{2}\right) \frac{\left[\left(a_2^{(U)}\right)^2 - a_1^{(U)} \left(1 - \frac{\lambda^2}{2}\right)\right]}{1 + \lambda a_1^{(U)} \left(1 - \frac{\lambda^2}{2}\right)^3} \lambda^8 \frac{v}{\sqrt{2}}, & m_c &\simeq \left[1 + \lambda a_1^{(U)} \left(1 - \frac{\lambda^2}{2}\right)^3\right] \frac{\lambda^4 v}{\sqrt{2}}, & m_t &\simeq \frac{v}{\sqrt{2}}, \\
m_d &= a_1^{(D)} \lambda^8 \frac{v}{\sqrt{2}}, & m_s &\simeq \frac{\left|a_2^{(D)} a_3^{(D)} - \left(a_4^{(D)}\right)^2\right|}{\sqrt{\left(a_3^{(D)}\right)^2 + \left(a_4^{(D)}\right)^2}} \lambda^5 \frac{v}{\sqrt{2}}, & m_b &\simeq \sqrt{\left(a_3^{(D)}\right)^2 + \left(a_4^{(D)}\right)^2} \lambda^3 \frac{v}{\sqrt{2}}.
\end{aligned} \tag{17}$$

The CKM quark mixing matrix is approximately given by:

$$V_{CKM} = R_U^T P R_D \simeq \begin{pmatrix} c & sc_D & ss_D \\ -sc_U & cc_{UCD} - s_U s_D e^{i\phi} & s_U c_D e^{i\phi} + cc_{USD} \\ ss_U & -c_U s_D e^{i\phi} - cs_U c_D & c_U c_D e^{i\phi} - cs_U s_D \end{pmatrix}, \tag{18}$$

where $c = \cos \theta$, $s = \sin \theta$, $c_{U,D} = \cos \theta_{U,D}$, $s_{U,D} = \sin \theta_{U,D}$ and the quark mixing angles are:

$$\sin \theta \simeq -\lambda, \quad \sin \theta_U \simeq -\frac{\lambda a_2^{(D)} \left(1 - \frac{\lambda^2}{2}\right)^3}{2a_1^{(D)}}, \quad \tan 2\theta_D = \frac{2 \left(a_2^{(D)} + a_3^{(D)}\right) a_4^{(D)}}{\left(a_3^{(D)}\right)^2 + \left(a_4^{(D)}\right)^2} \lambda^2. \tag{19}$$

It is noteworthy that Eqs. (17)-(19) provide an elegant understanding of all SM quark masses and mixing parameters in terms of the Wolfenstein parameter $\lambda = 0.225$ and of parameters of order unity. Note that all physical parameters in the quark sector are linked with the electroweak symmetry breaking scale $v = 246$ GeV through their scalings by powers of the Wolfenstein parameter $\lambda = 0.225$, with $\mathcal{O}(1)$ coefficients.

Observable	Model value	Experimental value
$m_u(MeV)$	1.11	$1.45_{-0.45}^{+0.56}$
$m_c(MeV)$	639	635 ± 86
$m_t(GeV)$	172.3	$172.1 \pm 0.6 \pm 0.9$
$m_d(MeV)$	2.9	$2.9_{-0.4}^{+0.5}$
$m_s(MeV)$	57.7	$57.7_{-15.7}^{+16.8}$
$m_b(GeV)$	2.82	$2.82_{-0.04}^{+0.09}$
$ V_{ud} $	0.974	0.97427 ± 0.00015
$ V_{us} $	0.22516	0.22534 ± 0.00065
$ V_{ub} $	0.00353	$0.00351_{-0.00014}^{+0.00015}$
$ V_{cd} $	0.22502	0.22520 ± 0.00065
$ V_{cs} $	0.97348	0.97344 ± 0.00016
$ V_{cb} $	0.0412	$0.0412_{-0.0005}^{+0.0011}$
$ V_{td} $	0.00860	$0.00867_{-0.00031}^{+0.00029}$
$ V_{ts} $	0.0404	$0.0404_{-0.0005}^{+0.0011}$
$ V_{tb} $	0.999145	$0.999146_{-0.000046}^{+0.000021}$
J	2.96×10^{-5}	$(2.96_{-0.16}^{+0.20}) \times 10^{-5}$
δ	68°	68°

Table III: Model and experimental values of the quark masses and CKM parameters.

To describe the quark masses and mixing, we have 9 parameters, i.e. λ , $a_1^{(U)}$, $a_2^{(U)}$, $a_{33}^{(U)}$, $a_3^{(D)}$, $a_4^{(D)}$, e_D , f_D and the phases ϕ , while the corresponding number of observables in the quark sector is 10. Note that the parameters λ and $a_{33}^{(U)}$ are fixed while the remaining 7 parameters are fitted to reproduce the 6 quark masses and 4 quark mixing

parameters. The results shown in Table III correspond to the following best-fit values:

$$\begin{aligned} a_1^{(U)} &\simeq 2.05, & a_2^{(U)} &\simeq 0.75, & a_1^{(D)} &\simeq 2.51, & a_2^{(D)} &\simeq -0.31, \\ a_3^{(D)} &\simeq 1.26, & a_4^{(D)} &\simeq -0.65, & \phi &\simeq -90.24^\circ, \end{aligned} \tag{20}$$

The obtained and experimental values of the observables in the quark sector are shown in Table III. The experimental values of the quark masses, which are given at the M_Z scale, have been taken from Ref. [49] (which are similar to those in [50]), whereas the experimental values of the CKM matrix elements, the Jarlskog invariant J and the CP violating phase δ are taken from Ref. [3]. As seen from Table III, the quark masses and CKM parameters are in excellent agreement with the experimental data. The agreement of our model with the experimental data is as good as in the models of Refs. [51–55], and better than many others approaches [29, 55–67].

IV. LEPTON MASSES AND MIXING

Using Eq. (10) it follows that the charged lepton mass matrix is given by:

$$M_l = \begin{pmatrix} a_{11}^{(l)} \lambda^8 & 0 & 0 \\ 0 & a_{22}^{(l)} \lambda^5 & a_{23}^{(l)} \lambda^3 \\ 0 & a_{32}^{(l)} \lambda^5 & a_{33}^{(l)} \lambda^3 \end{pmatrix} \frac{v}{\sqrt{2}}, \quad (21)$$

where:

$$\begin{aligned} a_{32}^{(l)} &= \frac{1}{v} (\alpha_{23} v_S - 6\beta_{23} v_\Phi), & a_{11}^{(l)} &= \frac{1}{v} (\alpha_{11} v_S - 6\beta_{11} v_\Phi), \\ a_{23}^{(l)} &= \frac{1}{v} (\alpha_{32} v_S - 6\beta_{32} v_\Phi), & a_{22}^{(l)} &= \frac{1}{v} (\alpha_{22} v_S - 6\beta_{22} v_\Phi), \\ a_{33}^{(l)} &= \frac{1}{v} (\alpha_{33} v_S - 6\beta_{33} v_\Phi), \end{aligned} \quad (22)$$

where the dimensionless couplings in Eq. (21) are $\mathcal{O}(1)$ parameters. From Eq. (14), it follows that the mass matrix for charged leptons can be rewritten as follows:

$$M_l = \begin{pmatrix} a_1^{(l)} \lambda^8 & 0 & 0 \\ 0 & a_2^{(l)} \lambda^5 & a_4^{(l)} \lambda^3 \\ 0 & a_3^{(l)} \lambda^5 & a_3^{(l)} \lambda^3 \end{pmatrix} \frac{v}{\sqrt{2}}. \quad (23)$$

The matrix $M_l M_l^T$ is diagonalized by a rotation matrix R_l according to:

$$\begin{aligned} R_l^T M_l M_l^T R_l &= \text{diag}(m_e, m_\mu, m_\tau), & R_l &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_l & -\sin \theta_l \\ 0 & \sin \theta_l & \cos \theta_l \end{pmatrix}, \\ \tan \theta_l &\simeq -\frac{a_4^{(l)}}{a_3^{(l)}}, & \cos \theta_l &\simeq \frac{a_3^{(l)}}{\sqrt{(a_3^{(l)})^2 + (a_4^{(l)})^2}}, & \sin \theta_l &\simeq -\frac{a_4^{(l)}}{\sqrt{(a_3^{(l)})^2 + (a_4^{(l)})^2}}, \end{aligned} \quad (24)$$

where, from Eq. (23) it follows that the charged lepton masses are approximately given by:

$$m_e = a_1^{(l)} \lambda^8 \frac{v}{\sqrt{2}}, \quad m_\mu \simeq \frac{|a_2^{(l)} a_3^{(l)} - (a_4^{(l)})^2|}{\sqrt{(a_3^{(l)})^2 + (a_4^{(l)})^2}} \lambda^5 \frac{v}{\sqrt{2}}, \quad m_\tau \simeq \sqrt{(a_3^{(l)})^2 + (a_4^{(l)})^2} \lambda^3 \frac{v}{\sqrt{2}}. \quad (25)$$

Note the remarkable feature that charged lepton masses are connected with the electroweak symmetry breaking scale $v = 246$ GeV through their power dependence on the Wolfenstein parameter $\lambda = 0.225$, with $\mathcal{O}(1)$ coefficients. This remarkable feature, which is also presented in the quark sector, is due to the fact that down type quarks and charged leptons are members of the same fermionic multiplets of the GUT group. Furthermore, it is noteworthy that unlike in the down type quark sector, the mixing angle θ_l in the charged lepton sector is large, which gives rise to an important contribution to the leptonic mixing matrix, coming from the mixing of charged leptons.

Since the four $SU(5)$ scalar singlets acquiring VEVs at the GUT scale have Yukawa interactions with the right handed Majorana neutrinos, the right handed Majorana neutrino masses have GUT scale values. Consequently, the entries of the diagonal heavy Majorana neutrino mass matrix satisfy $(M_R)_{ii} \gg v$, implying that the light neutrino masses are generated via type I seesaw mechanism. Then, from Eq. (10) it follows that the neutrino mass matrix is given by:

$$M_\nu = \begin{pmatrix} O_{3 \times 3} & M_\nu^D \\ (M_\nu^D)^T & M_R \end{pmatrix}, \quad M_\nu^D = \begin{pmatrix} A & D & 0 \\ B & E & 0 \\ C & F & 0 \end{pmatrix}, \quad M_R = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{pmatrix}, \quad (26)$$

where:

$$\begin{aligned} A &= \varepsilon_{11} \lambda^3 v_H^{(1)} \frac{v_\eta}{\Lambda}, & D &= \varepsilon_{12} \lambda^3 v_H^{(2)} \frac{v_\eta}{\Lambda}, & B &= \varepsilon_{21} \lambda^3 v_H^{(2)} \frac{v_\zeta}{\Lambda}, \\ E &= \varepsilon_{22} \lambda^3 v_H^{(1)} \frac{v_\zeta}{\Lambda}, & C &= \varepsilon_{31} \lambda^3 v_H^{(2)} \frac{v_\zeta}{\Lambda}, & F &= \varepsilon_{32} \lambda^3 v_H^{(1)} \frac{v_\eta}{\Lambda}, \\ M_i &= y_i \frac{v_\chi^2 + x_1 v_\sigma^2 + x_2 v_\eta^2 + x_3 v_\zeta^2}{\Lambda}, & i &= 1, 2, 3. \end{aligned} \quad (27)$$

Therefore the light neutrino mass matrix takes the following form:

$$M_L = M_\nu^D M_R^{-1} (M_\nu^D)^T = \begin{pmatrix} W^2 & WX \cos \varphi & WY \cos(\varphi - \varrho) \\ WX \cos \varphi & X^2 & XY \cos \varrho \\ WY \cos(\varphi - \varrho) & XY \cos \varrho & Y^2 \end{pmatrix}, \quad (28)$$

where:

$$\begin{aligned} W &= |\vec{W}| = \sqrt{\frac{A^2}{M_1} + \frac{D^2}{M_2}}, & X &= |\vec{X}| = \sqrt{\frac{B^2}{M_1} + \frac{E^2}{M_2}}, & Y &= |\vec{Y}| = \sqrt{\frac{C^2}{M_1} + \frac{F^2}{M_2}}, \\ \vec{W} &= \left(\frac{A}{\sqrt{M_1}}, \frac{D}{\sqrt{M_2}} \right), & \vec{X} &= \left(\frac{B}{\sqrt{M_1}}, \frac{E}{\sqrt{M_2}} \right), & \vec{Y} &= \left(\frac{C}{\sqrt{M_1}}, \frac{F}{\sqrt{M_2}} \right), \\ \cos \varphi &= \frac{\vec{W} \cdot \vec{X}}{|\vec{W}| |\vec{X}|}, & \cos(\varphi - \varrho) &= \frac{\vec{W} \cdot \vec{Y}}{|\vec{W}| |\vec{Y}|}, & \cos \varrho &= \frac{\vec{X} \cdot \vec{Y}}{|\vec{X}| |\vec{Y}|}. \end{aligned} \quad (29)$$

To simplify the analysis, we set $\varphi = \varrho$, obtaining that the light neutrino mass matrix is given by:

$$M_L = \begin{pmatrix} W^2 & \kappa WX & WY \\ \kappa WX & X^2 & \kappa XY \\ WY & \kappa XY & Y^2 \end{pmatrix}, \quad \kappa = \cos \varphi. \quad (30)$$

Assuming that the neutrino Yukawa couplings are real, we find that for the normal (NH) and inverted (IH) mass hierarchies, the light neutrino mass matrix is diagonalized by a rotation matrix R_ν , according to:

$$\begin{aligned} R_\nu^T M_L R_\nu &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & m_{\nu_2} & 0 \\ 0 & 0 & m_{\nu_3} \end{pmatrix}, & R_\nu &= \begin{pmatrix} -\frac{Y}{\sqrt{W^2+Y^2}} & \frac{W}{\sqrt{W^2+Y^2}} \sin \theta_\nu & \frac{W}{\sqrt{W^2+Y^2}} \cos \theta_\nu \\ 0 & \cos \theta_\nu & -\sin \theta_\nu \\ \frac{W}{\sqrt{W^2+Y^2}} & \frac{Y}{\sqrt{W^2+Y^2}} \sin \theta_\nu & \frac{Y}{\sqrt{W^2+Y^2}} \cos \theta_\nu \end{pmatrix}, & \text{for NH} \end{aligned} \quad (31)$$

$$\tan \theta_\nu = -\sqrt{\frac{m_3 - X^2}{X^2 - m_2}}, \quad m_{\nu_1} = 0, \quad m_{\nu_{2,3}} = \frac{W^2 + X^2 + Y^2}{2} \mp \frac{\sqrt{(W^2 - X^2 + Y^2)^2 - 4\kappa^2 X^2 (W^2 + Y^2)}}{2}.$$

$$\begin{aligned} R_\nu^T M_L R_\nu &= \begin{pmatrix} m_{\nu_1} & 0 & 0 \\ 0 & m_{\nu_2} & 0 \\ 0 & 0 & 0 \end{pmatrix}, & R_\nu &= \begin{pmatrix} \frac{W}{\sqrt{W^2+Y^2}} & -\frac{Y}{\sqrt{W^2+Y^2}} \sin \theta_\nu & -\frac{Y}{\sqrt{W^2+Y^2}} \cos \theta_\nu \\ 0 & \cos \theta_\nu & -\sin \theta_\nu \\ \frac{Y}{\sqrt{W^2+Y^2}} & \frac{W}{\sqrt{W^2+Y^2}} \sin \theta_\nu & \frac{W}{\sqrt{W^2+Y^2}} \cos \theta_\nu \end{pmatrix}, & \text{for IH} \end{aligned} \quad (32)$$

$$\tan \theta_\nu = -\sqrt{\frac{m_2 - X^2}{X^2 - m_1}}, \quad m_{\nu_{1,2}} = \frac{W^2 + X^2 + Y^2}{2} \mp \frac{1}{2} \sqrt{(W^2 - X^2 + Y^2)^2 - 4\kappa^2 X^2 (W^2 + Y^2)}, \quad m_{\nu_3} = 0.$$

It is noteworthy that the smallness of the active neutrinos masses is a consequence of their scaling with the inverse of the large Majorana neutrino masses, as expected from the type I seesaw mechanism implemented in our model.

With the rotation matrices in the charged lepton sector R_l , given by Eq. (24), and in the neutrino sector R_ν , given by Eqs. (31) and (32) for NH and IH, respectively, we find the PMNS mixing matrix:

$$U = R_l^T R_\nu = \begin{cases} \begin{pmatrix} -\frac{Y}{\sqrt{W^2+Y^2}} & \frac{W}{\sqrt{W^2+Y^2}} \sin \theta_\nu & \frac{W}{\sqrt{W^2+Y^2}} \cos \theta_\nu \\ \frac{W}{\sqrt{W^2+Y^2}} \sin \theta_l & \cos \theta_l \cos \theta_\nu + \frac{Y}{\sqrt{W^2+Y^2}} \sin \theta_l \sin \theta_\nu & \frac{Y}{\sqrt{W^2+Y^2}} \cos \theta_\nu \sin \theta_l - \cos \theta_l \sin \theta_\nu \\ \frac{W}{\sqrt{W^2+Y^2}} \cos \theta_l & \frac{Y}{\sqrt{W^2+Y^2}} \cos \theta_l \sin \theta_\nu - \cos \theta_\nu \sin \theta_l & \sin \theta_l \sin \theta_\nu + \frac{Y}{\sqrt{W^2+Y^2}} \cos \theta_l \cos \theta_\nu \end{pmatrix} & \text{for NH,} \\ \begin{pmatrix} \frac{W}{\sqrt{W^2+Y^2}} & -\frac{Y}{\sqrt{W^2+Y^2}} \sin \theta_\nu & -\frac{Y}{\sqrt{W^2+Y^2}} \cos \theta_\nu \\ \frac{Y}{\sqrt{W^2+Y^2}} \sin \theta_l & \cos \theta_l \cos \theta_\nu + \frac{W}{\sqrt{W^2+Y^2}} \sin \theta_\nu \sin \theta_l & \frac{W}{\sqrt{W^2+Y^2}} \sin \theta_l \cos \theta_\nu - \cos \theta_l \sin \theta_\nu \\ \frac{Y}{\sqrt{W^2+Y^2}} \cos \theta_l & \frac{W}{\sqrt{W^2+Y^2}} \sin \theta_\nu \cos \theta_l - \cos \theta_\nu \sin \theta_l & \sin \theta_l \sin \theta_\nu + \frac{W}{\sqrt{W^2+Y^2}} \cos \theta_l \cos \theta_\nu \end{pmatrix} & \text{for IH.} \end{cases} \quad (33)$$

From the standard parametrization of the leptonic mixing matrix, it follows that the lepton mixing angles for NH and IH, respectively, are:

$$\begin{aligned} \sin^2 \theta_{12} &= \frac{W^2 \sin^2 \theta_\nu}{Y^2 + (1 - \cos^2 \theta_\nu) W^2}, & \sin^2 \theta_{13} &= \frac{W^2 \cos^2 \theta_\nu}{W^2 + Y^2}, \\ \sin^2 \theta_{23} &= \frac{(\sqrt{W^2 + Y^2} \sin \theta_\nu \cos \theta_l - Y \cos \theta_\nu \sin \theta_l)^2}{(1 - \cos^2 \theta_\nu) W^2 + Y^2}, & & \text{for NH} \end{aligned} \quad (34)$$

$$\begin{aligned} \sin^2 \theta_{12} &= \frac{Y^2 \sin^2 \theta_\nu}{W^2 + (1 - \cos^2 \theta_\nu) Y^2}, & \sin^2 \theta_{13} &= \frac{Y^2 \cos^2 \theta_\nu}{W^2 + Y^2}, \\ \sin^2 \theta_{23} &= \frac{(\sqrt{W^2 + Y^2} \sin \theta_\nu \cos \theta_l - W \cos \theta_\nu \sin \theta_l)^2}{(1 - \cos^2 \theta_\nu) Y^2 + W^2}, & & \text{for IH} \end{aligned} \quad (35)$$

Varying the lepton sector model parameters $a_i^{(l)}$ ($i = 1, 2, 3, 4$), κ , W , X and Y , we fitted the charged lepton masses, the neutrino mass squared splittings Δm_{21}^2 , Δm_{31}^2 (note that we define $\Delta m_{ij}^2 = m_i^2 - m_j^2$) and the leptonic mixing angles $\sin^2 \theta_{12}$, $\sin^2 \theta_{13}$ and $\sin^2 \theta_{23}$ to their experimental values for NH and IH. The results shown in Table IV correspond to the following best-fit values:

$$\begin{aligned} \kappa &\simeq 0.45, & W &\simeq 0.13eV^{\frac{1}{2}}, & X &\simeq 0.11eV^{\frac{1}{2}}, & Y &\simeq 0.18eV^{\frac{1}{2}}, \\ a_1^{(l)} &\simeq 0.42, & a_2^{(l)} &\simeq 1.39, & a_3^{(l)} &\simeq 0.77, & a_4^{(l)} &\simeq 0.42, & \text{for NH} \end{aligned} \quad (36)$$

$$\begin{aligned} \kappa &\simeq 4.03 \times 10^{-3}, & W &\simeq 0.18eV^{\frac{1}{2}}, & X &\simeq 0.22eV^{\frac{1}{2}}, & Y &\simeq 0.13eV^{\frac{1}{2}}, \\ a_1^{(l)} &\simeq 0.42, & a_2^{(l)} &\simeq 1.38, & a_3^{(l)} &\simeq 0.78, & a_4^{(l)} &\simeq 0.42, & \text{for IH} \end{aligned} \quad (37)$$

Using the best-fit values given above, we get for NH and IH, respectively, the following neutrino masses:

$$m_1 = 0, \quad m_2 \approx 9\text{meV}, \quad m_3 \approx 50\text{meV}, \quad \text{for NH} \quad (38)$$

$$m_1 \approx 49\text{meV}, \quad m_2 \approx 50\text{meV}, \quad m_3 = 0, \quad \text{for IH} \quad (39)$$

The obtained and experimental values of the observables in the lepton sector are shown in Table IV. The experimental values of the charged lepton masses, which are given at the M_Z scale, have been taken from Ref. [49] (which are

Observable	Model value	Experimental value
$m_e(MeV)$	0.487	0.487
$m_\mu(MeV)$	102.8	102.8 ± 0.0003
$m_\tau(GeV)$	1.75	1.75 ± 0.0003
$\Delta m_{21}^2(10^{-5}eV^2)$ (NH)	7.60	$7.60^{+0.19}_{-0.18}$
$\Delta m_{31}^2(10^{-3}eV^2)$ (NH)	2.48	$2.48^{+0.05}_{-0.07}$
$\sin^2 \theta_{12}$ (NH)	0.323	0.323 ± 0.016
$\sin^2 \theta_{23}$ (NH)	0.567	$0.567^{+0.032}_{-0.128}$
$\sin^2 \theta_{13}$ (NH)	0.0234	0.0234 ± 0.0020
$\Delta m_{21}^2(10^{-5}eV^2)$ (IH)	7.60	$7.60^{+0.19}_{-0.18}$
$\Delta m_{13}^2(10^{-3}eV^2)$ (IH)	2.38	$2.48^{+0.05}_{-0.06}$
$\sin^2 \theta_{12}$ (IH)	0.323	0.323 ± 0.016
$\sin^2 \theta_{23}$ (IH)	0.0573	$0.573^{+0.025}_{-0.043}$
$\sin^2 \theta_{13}$ (IH)	0.0240	0.0240 ± 0.0019

Table IV: Model and experimental values of the charged lepton masses, neutrino mass squared splittings and leptonic mixing parameters for the normal (NH) and inverted (IH) mass hierarchies.

similar to those in [50]), whereas the experimental values of the neutrino mass squared splittings and leptonic mixing angles for both normal (NH) and inverted (IH) mass hierarchies, are taken from Ref. [9]. The obtained charged lepton masses, neutrino mass squared splittings and lepton mixing angles are in excellent agreement with the experimental data for both normal and inverted neutrino mass hierarchies. Let us remind that for the sake of simplicity, we assumed all leptonic parameters to be real, but a non-vanishing CP violating phase in the PMNS mixing matrix can be generated by making one of the entries of the neutrino mass matrix of Eq. (26) to be complex.

V. CONCLUSIONS

We proposed a model based on the $SU(5)$ grand unification with an extra $Z_2 \otimes Z'_2 \otimes Z''_2 \otimes Z_4 \otimes Z_{12}$ flavor symmetry, that successfully accounts for the SM fermion masses and mixings. The model has in total 17 effective free parameters, from which 2 are fixed and 15 are fitted to reproduce the experimental values of 18 observables, i.e., 9 charged fermion masses, 2 neutrino mass squared splittings, 3 lepton mixing parameters, 3 quark mixing angles and 1 CP violating phase of the CKM quark mixing matrix. One of the two fixed parameters is identified with the Wolfenstein one and the other one is set to one as suggested by the naturalness arguments. The observed quark mass and mixing hierarchy is caused by the Z_4 and Z_{12} symmetries, which are broken at very high scale by the $SU(5)$ scalar singlets σ and χ , respectively charged under these symmetries, and which acquire VEVs at the GUT scale. The active neutrino masses of the model are generated via type I seesaw mechanism with three heavy Majorana neutrinos. The smallness of the active neutrino masses is attributed to their scaling with inverse powers of the large Majorana neutrino masses. The model predictions for the observables in both quark and lepton sectors are in excellent agreement with the experimental data.

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